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NUMERICAL ANALYSIS OF SOILS IN SIMPLE SHEAR DEVICES

MUNIRAM BUDHU* and ARUL BRITTO**

ABSTRACT

The results of finite element analyses of soils in simple shear devices, assuming that these soils can be modelled either as an elastic or an elastoplastic material, are presented. The results indicate that an elastic analysis produces larger levels of stress concentrations than an analysis using the modified Cam-clay model. The predicted stress-strain behavior of a very loose sand and speswhite kaolin using the modified Cam-clay model agrees very well with simple shear test results deduced from measurements made at the sample core of the top boundary of the samples for constant load tests. A satisfactory match of experimental and modified Cam-clay stress-strain results was not obtained for the constant height test. The simple shear devices (Norwegian Geotechnical Institute and Cambridge types) can be expected to give good quality results for monotonic loading from carefully prepared samples if measurements of stress and pore water pressures are made at the sample core on either the top or bottom or both horizontal boundaries of the sample.

Key words: clay, deformation, direct shear test, sand, stress distribution, stress-strain curve, (non-uniformities) (**IGC**: D 6)

INTRODUCTION

Laboratory apparatus which can impose a state of plane strain and allow the rotation of the principal axes of stress are desirable since this condition appears to simulate the stress state to which soils are subjected in many practical situations. For example, soils adjacent to a friction pile or beneath the foundation of an offshore platform can be expected to deform in a manner similar to simple shear strain. Simple shear strain is a form of plane strain in which an initially cuboidal element is transformed into a parallelepiped without change in volume as shown in Fig. 1. For a uniform stress state to occur under simple shear strain, equilibrium demands that complementary shear stresses be developed on the vertical sides of the element normal to the plane of deformation.

Two apparatus with different purposes have been developed to supposedly impose simple shear strain to soil samples. One, originally proposed by Roscoe (1953) for research and

* Assistant Professor, Dept. of Civil Engineering, State University of New York at Buffalo, Buffalo, NY 14260, USA.
** Senior Research Associate, University of Cambridge, Cambridge, UK, formerly Visiting Assistant Professor, SUNYAB, NY 14260, USA.
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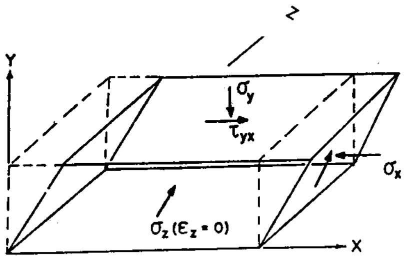


Fig. 1. Simple shear strain

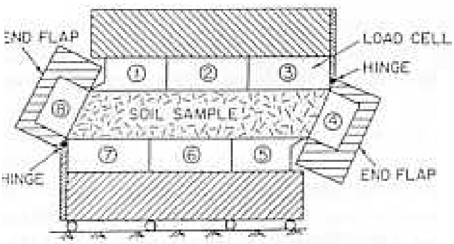


Fig. 2. Cambridge simple shear apparatus

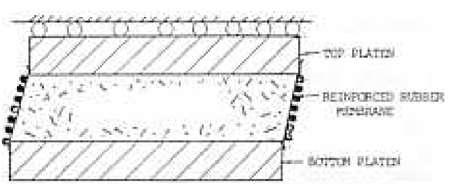


Fig. 3. Norwegian Geotechnical Institute apparatus

continuously upgraded at Cambridge University, tests a cuboidal sample (100 mm × 100 mm × 20 mm high) between rigid boundaries (Fig. 2). Simple shear strain is applied through two hinged end flaps which rotate when the bottom boundary of the device is horizontally displaced. The inner walls of these end flaps are smooth so that significant shear stresses cannot be generated there. The other, originally proposed by Kjellman (1951) and modified by Bjerrum and Landva (1966) at the Norwegian Geotechnical Institute (NGI) for practical use, tests a cylindrical sample (80 mm diameter × 20 mm high) laterally confined by a wire reinforced membrane between rigid top and bottom platens (Fig. 3). Simple shear strain is presumed to be imposed by displacing the top boundary.

Neither of these apparatus allows the development of complementary shear stresses on the vertical sides normal to the plane of deformation. As a result, the shear and normal stresses must be non-uniformly distributed to satisfy equilibrium (Wood, Drescher and Budhu, 1979 ; Budhu, 1984 ; Airey, Budhu and Wood, 1985).

In order to evaluate the practical effects and actual distribution of normal and shear stresses, studies have proceeded along four different directions—analytical / numerical (Roscoe, 1953 ; Duncan and Dunlop, 1969 ; Lucks et al., 1972 ; Prevost and Hoeg, 1976 ; Hara and Kyota, 1977 ; Shen et al., 1978) ; experimental (Cole, 1967 ; Stroud, 1971 ; Budhu, 1979 ; Airey, 1984) ; comparisons of simple shear test results with results from either the triaxial or hollow cylinder apparatus or both (Saada et al., 1983) ; and discussion (Vucetic and Lacasse, 1982).

In the analytical/numerical studies, elastic material behavior was assumed and it was shown that for the Cambridge simple shear apparatus (SSA) at least the middle one-third of the sample and for the NGI SSA, seventy percent of the sample could be expected to deform uniformly. Detailed experimental work in these devices is tedious and require sophisticated instrumentation and as such cannnot be conducted on a routine basis. However, such tests in sands by Stroud (1971) and Budhu (1979, 1984) did show that the middle one-third of the samples in monotonic tests deform uniformly in these devices but that non-uniformities predominant at the ends spread rapidly during cyclic loading. Airey (1984), using a modified version of a specially instrumented NGI type device originally developed for sand by Budhu (1979), showed that the normal stresses are much more uniform for clays than for sands.

Comparisons of simple shear test results with results from other devices—the triaxial and hollow cylinder apparatus—show that the simple shear devices produce lower soil strengths and stiffnesses and as a result simple shear devices have been dismissed (Saada and Townsend, 1981). There is no obvious reason why simple shear test results should agree with results from triaxial or other devices unless the stress paths followed are identical and the stresses imposed produce similar changes in soil fabric.

In comparing results from various testing devices, it is necessary to select appropriate stress and strain parameters. In simple shear tests, the intermediate principal stress (σ_2) is neither independent of nor is it equal to either the major or minor principal stresses. Consequently, the effect of the intermediate stresses should not be ignored as a rule.

Recent tests on clays (Airey, 1984) in an elaborately instrumented NGI type simple shear device show that as the plasticity of the material increases so does the uniformity of boundary stresses. It appears that elastic analyses produce results which perhaps show up the non-uniformities in simple shear devices too pessimistically. Of course, soil is neither isotropic nor elastic. It is the intention of this paper to examine the distribution of stresses and the mechanical behavior of a sand and a clay in the simple shear devices assuming these soils behave as either elastic or elastoplastic materials.

ELASTOPLASTIC SOIL MODEL

The model used is the modified Cam-clay model which has been shown to predict the mechanical behavior of normally and lightly overconsolidated soils (overconsolidation ratio less than 2) reasonably well (see for example, Wroth, 1977). The details of this model have been elaborated on by others (for example, Roscoe and Burland, 1968 ; Wroth and Houlsby, 1985) and need not be attempted here. Essentially, the model consists of an ellipsoidal yield surface in p, q space (Fig. 4(a)) where :

$$p=(1/3)\ \delta_{ij}\sigma_{ij} \tag{1}$$

ıd

$$q=(3/2\ S_{ij}S_{ij})^{1/2} \tag{2}$$

p is the mean effective stress
q is the deviatoric stress

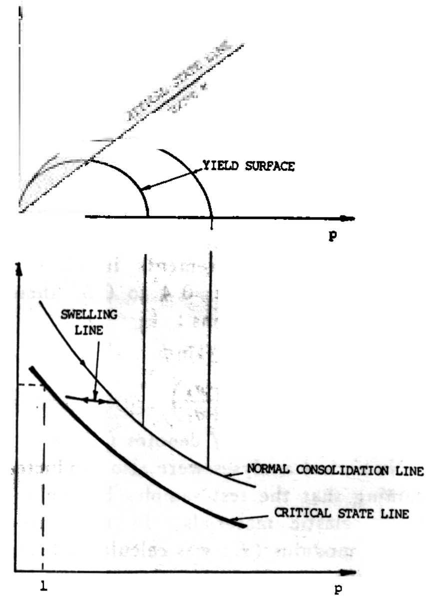


Fig. 4. Modified cam-clay yield surface and *e*-*p* relationship

δ_{ij} is Kronecker delta
σ_{ij} is the effective stress tensor
S_{ij} is the stress deviator

During loading the yield surface expands and this expansion is linked to the isotropic normal consolidation line (Fig. 4 (b)). The soil parameters required for this model are:

λ —the slope of the consolidation line
κ —the slope of the swelling line
ϕ or M—the effective angle of friction or the frictional constant at failure in (q, p) space respectively
G or μ—the shear modulus or Poisson's ratio
e_λ —void ratio at unit effective mean stress on the critical state line.

For simple shear strain tests, a knowledge of either the magnitude of the intermediate principal stress or its relationship to the other two principal stresses is needed to determine M. Unfortunately, the intermediate principal stresses in simple shear devices are not routinely measured or cannot be measured as is the case for the NGI SSA.

Drained tests in the Cambridge SSA conducted by Stroud (1971) and Budhu (1979) on 14/25 Leighton Buzzard (LB) sand (average grain size 1 mm) showed that

$$\sigma_2 = m(\sigma_1 + \sigma_3) \qquad (3)$$

where m is a constant, σ_1 and σ_3 are the major and minor effective principal stresses. For 14/25 Leighton Buzzard sand $m=0.37$ (Stroud, 1971). The authors are not aware of any such measurements in clays. Assuming values of $m=0.4$ to 0.5, then for simple shear conditions:

$$M \simeq \sqrt{3}\sin\phi \qquad (4)$$

where $\sin\phi = \left(\frac{\sigma_1 - \sigma_3}{\sigma_1 + \sigma_3}\right)_f$

and the subscript f denotes failure.

Numerical analyses were also conducted assuming that the test samples behave as isotropic elastic materials. In this case the elastic modulus (E) was calculated from the expression:

$$E = \frac{3p(1+e)(1-2\mu)}{\kappa} \qquad (5$$

where e is void ratio

SIMPLE SHEAR TESTS

In conventional simple shear tests, the soil sample is consolidated under K_0 condition by applying a vertical stress and then shear displacements are subsequently imposed. Two types of tests are often carried out. One, called the constant load test, permits the passage of water from the sample through porous stones placed on either the top or bottom boundaries or both, and the top boundary is allowed to move vertically to follow the compression or the dilation of the test specimens. Constant load tests on sands are generally carried out on dry material. The other, called the constant height test, is similar to the constant load test but the height between the top and bottom platens is kept constant. The reduction or increase in vertical stress required to maintain constant sample height is then related to the pore water pressures that might be developed in an undrained test under constant vertical load.

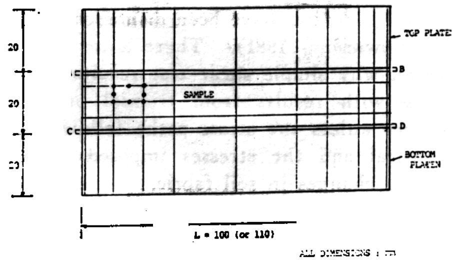


Fig. 5 (a). **Finite Element Mesh used**

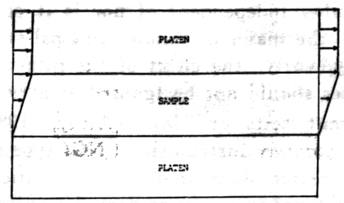


Fig. 5 (b). **Displacements imposed on vertical sides**

FINITE ELEMENT ANALYSIS

CRISP (Gunn and Britto, 1984) is a finite element program developed at Cambridge University. The program allows prediction of plane strain, axisymmetry and three-dimensional boundary value problems. Constitutive models included in the program are the critical state models (Cam-clay, modified Cam-clay) elastic and elastic perfectly plastic models. The modified Cam-clay and the elastic models are used in the present work.

A typical mesh used is shown in Fig. 5(a). The mesh includes the top and bottom metallic platens. These platens are assumed to behave elastically. Each element is an eight noded linear strain quadrilateral and the displacements are assumed to be quadratic functions of position coordinates. A 3×3 integration scheme was employed.

The constant load test is simulated by firstly consolidating the sample and then prescribing horizontal displacements on the top platen with the bottom platen fixed. The top platen is permitted to move vertically. The vertical sides are assumed to be on rollers. The nodes on the vertical sides are displaced following a uniform triangular distribution as shown in Fig. 5(b).

The constant height test is simulated in the same manner as the constant load test except that the top platen is prevented from moving vertically.

The displacements are applied in small increments and the nodal coordinates are updated after each increment. Equilibrium is, therefore, satisfied in the final (deformed) configuration. The updated coordinate scheme adopted here is not a rigorous treatment of finite deformation analyses. But, since significant displacements are imposed on the test samples (final shear strains are about 10 percent), this scheme is preferred to the alternative of satisfying equilibrium with respect to the original configuration.

The effect of the stiffness of the reinforced membrane of the NGI SSA is not included in this study. Shen et al (1978) using isotropic elastic material behavior showed that the uniformity of stresses and strains improves by increasing the stiffness of the reinforced membrane. Budhu (1979) showed experimentally, for LB sand tested in an NGI type SSA, that stiffer reinforcing wires improves the uniformity of stresses. Vucetic and Lacasse (1982) reported that the use of thicker than standard membranes (thickness varies from 0.56 mm to 0.67 mm) resulted in lower shear strengths and moduli.

The simulation used here is intended to represent conditions on the principal third of the test samples in both the Cambridge and NGI type SSA (see Fig. 6). For the Cambridge SSA, the size of the test samples analyzed is 100 mm × 100 mm × 20 mm high and for the NGI type SSA, the sample size is 110 mm diameter × 20 mm high. The mesh dimensions are chosen to make direct comparison with results from specially instrumented tests conducted by Budhu (1979) and Airey (1984). Even though it may seem inappropriate to compare the test results from the NGI type SSA apparatus with a plane strain analysis, it is reasonable to make such a comparison for the core

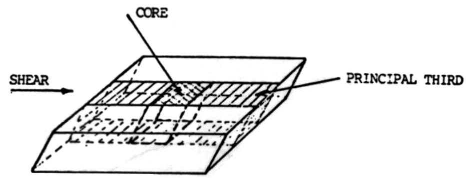


Fig. 6. Principal third and sample core

part of the sample.

MATERIAL PROPERTIES

It is assumed that the platens are made out of aluminium and the test samples are either 14/25 Leighton Buzzard sand or Speswhite kaolin. The following properties were selected.

Platens : $E=65\times10^6$ kN/m², $G=25\times10^6$ kN/m², $\mu=0.33$ (Roark and Young, 1975)

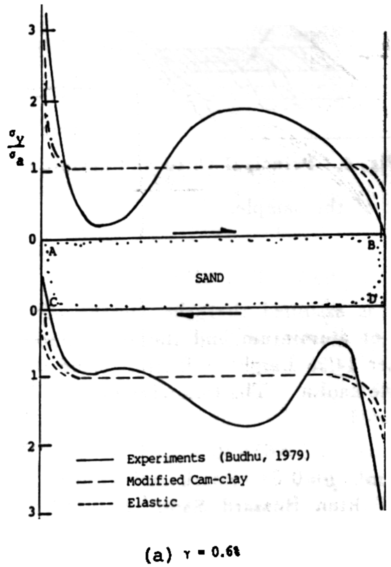
Leighton Buzzard Sand : $\lambda=0.025$ (Airey et al., 1985), $\kappa=0.005$, $\phi=35°$ (Stroud, 1971), $e_\lambda=0.927$ (Airey et al., 1985), $\mu=0.37$ (Stroud, 1971), $K_0=0.425$ (Budhu, 1979)

Speswhite Kaolin (PI=35%) : $\lambda=0.25$ (Lawrence, 1980), $\kappa=0.05$, $\phi=21°$ (Nadarajah, 1973), $e_\lambda=2.5$, $\mu=0.33$, $K_0=0.685$ (Airey, 1984)

COMPARISON OF NUMERICAL AND EXPERIMENTAL RESULTS

Distribution of Vertical Stresses

The vertical stress distribution on the top and bottom boundaries for very loose (relative density≃7 percent) LB sand and Speswhite kaolin obtained from the numerical analyses for constant load tests are compared with those from the experimental observations in Figs. 7 and 8. All the results reported in this paper for LB sand and Speswhite kaolin were obtained from tests carried out in the Cambridge SSA (Budhu, 1979) and a specially instrumented NGI type SSA (Airey, 1984) respectively. The experimental vertical stress distributions were deduced from load cells placed on the boundaries of these devices (Budhu, 1984). It is assumed here that the centroidal vertical stresses in



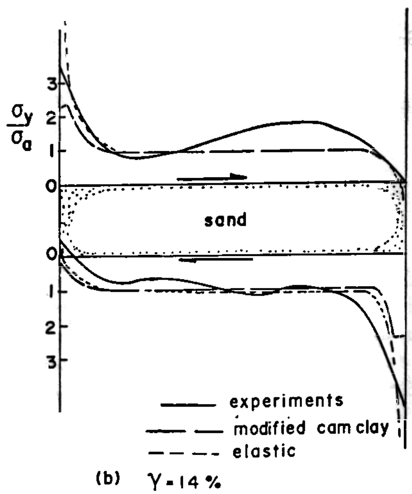


Fig. 7. Vertical stress distribution on loose sand (constant load test)

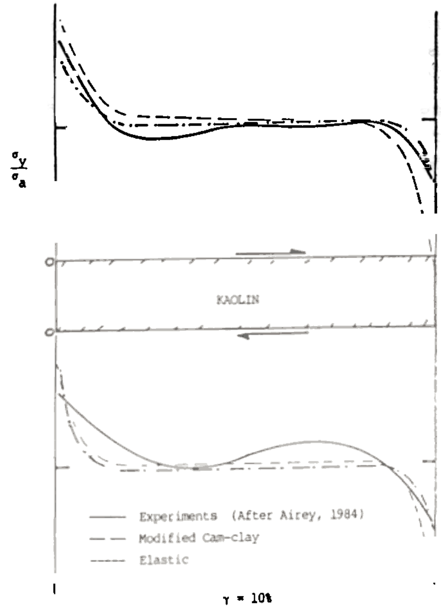


Fig. 8. Vertical stress distribution on kaolin (constant load test)

the elements in the first and last rows (AB and CD, 1 mm thick, Fig. 5) of the sample obtained from the numerical analysis are appropriate for comparisons with the experimental results. The vertical stresses (σ_y) are all normalized with respect to the applied vertical stress σ_a. By default all stresses are effective stresses. For LB sand the latter value is 100 kN/m^2 and for kaolin it is 200 kN/m^2.

At low shear strain levels where elastic behavior is expected to be predominant the modified Cam-clay and elastic analyses give sensibly the same results for LB sand (Fig. 7(a)). However, there are significant differences between the numerical and experimental results at this strain level. The LB sand used in the experiments was in a very loose state and during the consolidation phase very small lateral movements of the hinged side walls of the Cambridge SSA was sufficient to cause a collapse of the loose sand structure in the vicinity of these side walls. Thus, the differences between the numerical and experimental results may at least be partially attributed to the non-uniformities inherited during the consolidation phase. At high strain levels (Figs. 7(b) and 8) where plastic behavior is predominant, the elements

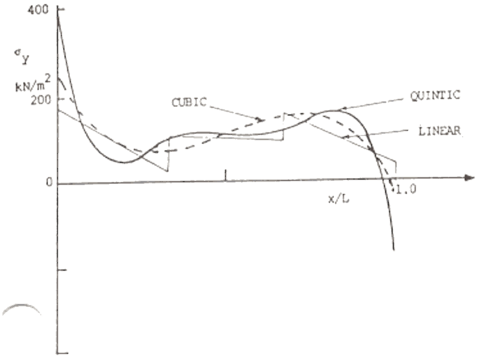


Fig. 9. Comparison of quintic, cubic and linear vertical stress

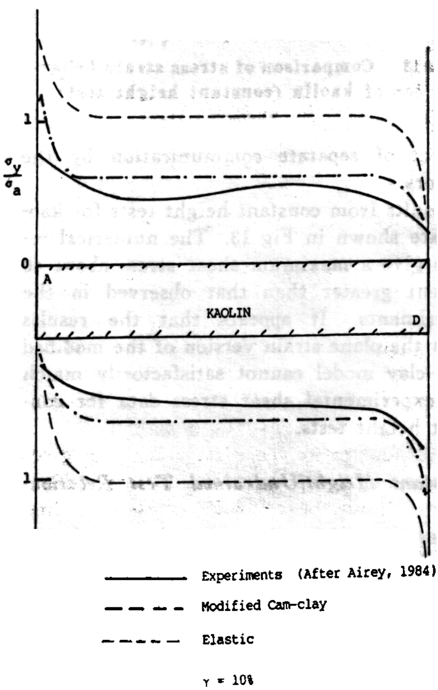


Fig. 10. Vertical stress distribution on kaolin (constant height)

close to the ends A and D (Fig. 5(a)) which are subjected to very high vertical stresses yield quickly reaching critical state, and transfer stresses to adjacent elements. The elastic analyses, as expected, continuously show very high stresses at these ends.

The experimental vertical stress distributions were obtained by fitting the information generated by load cells placed on the top and bottom boundaries by a quintic polynomial (Budhu, 1984). Other forms of distributions are possible. For example, a cubic polynomial can be fitted to the data using the least square method or a simple linear distribution can be used as done by Stroud (1971). A typical comparison of these distributions is shown in Fig. 9 where it is seen that the cubic polynomial produces less wiggles than the quintic polynomial (as expected). The linear stress distribution shows sudden changes in stresses which cannot, in practice, occur. The quintic polynomial distribution also shows the development of tensile stresses which are unsustainable by soils.

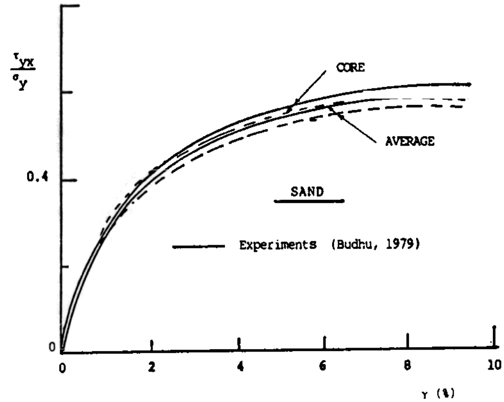


Fig. 11. Comparison of stress strain behavior of LB sand (constant vertical load)

Fig. 10 shows the vertical stress distribution for a constant height test on kaolin at $\gamma = 10\%$. The agreement between the experimental and analytical results at this rather large strain level is encouraging. Again, the results from the elastic analyses show much larger vertical stresses at ends A and D than those obtained from the experiments and the modified Cam-clay model.

Stress-strain Behavior

Fig. 11 shows the comparison of the nor-

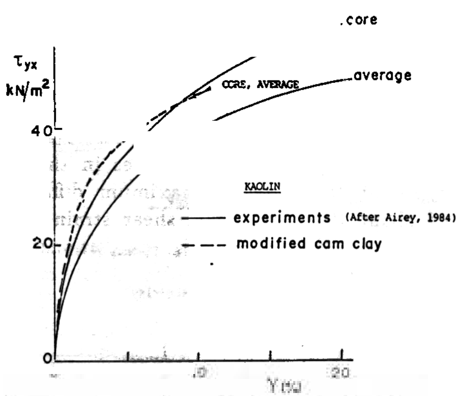


Fig. 12. Comparison of stress strain behavior of kaolin (constant load test)

malized shear stress ratio—shear strain response of LB sand if measurements were made at the sample core (Fig. 6) and over the top boundary for a constant load test. The latter measurements are those normally made in routine tests. The experimental and modified Cam-clay results show good agreement. Both results show a difference of shear stress ratio about 4 percent between the sample core and the average on the top boundary.

A similar comparison for kaolin, except now the shear stress on the horizontal boundary of the sample is reported (Airey, 1984) rather than the normalized shear stress ratio, is shown in Fig. 12. The numerical results give no significant difference in shear stress between the sample core and the average over the top boundary up to about 10 percent shear strain whereas the experimental results show a difference of about 16 percent. The numerical analysis did not account for slippage between the platens and soils. Although techniques have been developed to minimize slippage (Stroud, 1971; Budhu, 1984 ; Airey, 1984) some slippage at large strains could be expected during experiments using the simple shear devices. To account for slippage at platens-soil interface requires the use of slip elements in the numerical analysis. This would be the

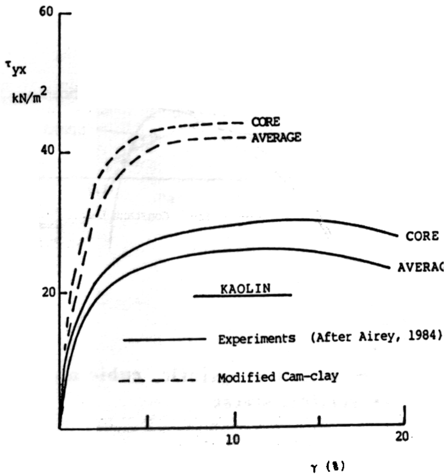


Fig. 13. Comparison of stress strain behavior of kaolin (constant height test)

subject of separate communication by the authors.

Results from constant height tests for kaolin are shown in Fig. 13. The numerical results give a maximum shear stress about 40 percent greater than that observed in the experiments. It appears that the results from the plane strain version of the modified Cam-clay model cannot satisfactorily match the experimental shear stress data for constant height tests.

Constant Height/Undrained Test Relationship

Bjerrum and Landva (1966) proposed that the constant height test in the NGI device is similar to an undrained test and that the changes in vertical stresses required to maintain constant height are equivalent to the changes in pore water pressures generated in a constant load test if drainage of water from the samples were prevented. There is controversy on this proposal. Saada et al. (1983) reported results from simple shear (NGI type device) and hollow cylinder tests on Edgar Plastic Kaolin and Reid Bedford sand which show that the changes in vertical stresses in constant height tests are not equiv-

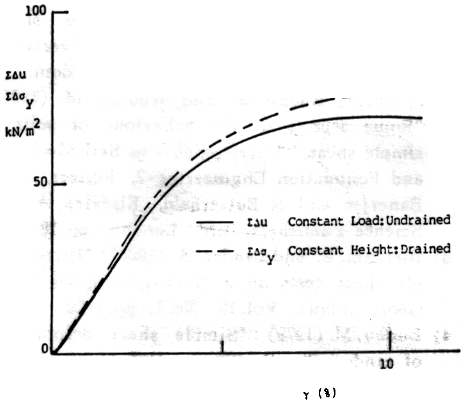


Fig. 14. Comparison of changes in vertical stresses with changes in pore water pressures

alent to the pore water pressures in constant load tests. They found that none of their normalization procedures was able to bring the two results to equivalency. However, Vucetic and Lacasse (1984) presented results from anisotropically consolidated triaxial tests on Drammen clay which are contrary to the results reported by Saada et al. (1983).

Saada et al. (1983) were only able to measure the average vertical stresses on presumably the top boundary of their samples and it is not clear where pore water pressure measurements were made. There is no obvious reason to suppose that the triaxial test results presented by Vucetic and Lacasse (1984) would be true for soils tested in the simple shear devices. It seems intuitively plausible, however, that if a soil sample in the simple shear devices were to deform as a single element, then there should be some relationship between the changes in vertical stresses in a constant height test and the changes in pore water pressure which would occur in a constant load test under undrained conditions.

Fig. 14 shows the finite element results of the changes in vertical stresses $\Delta\sigma_y$ and pore water pressures (Δu) from a constant height test and a constant load test (undrained conditions) respectively on the top 1 mm layer at the sample core (Figs. 5(a) and 6). These results represent predictions using the modified Cam-clay model and the properties of speswhite kaolin presented earlier in this paper.

The changes in vertical stresses and changes in pore water pressures in the two types of test show a maximum difference of about 10 percent at a shear strain of 10 percent. Of course, these predictions will vary depending on the choice of soil model. But, it is clear from Fig. 14 that if the sample were to behave as a single element (as demonstrated for the sample core by analysis and experiments) then one could expect (for all practical purposes) that the changes in vertical stresses in a constant height test would be directly related to the changes in pore water pressures in a constant load test conducted under undrained conditions. However, good quality simple shear tests in which the pore water pressures and stresses are measured on the sample core are required to further validate Bjerrum and Landva's supposition.

CONCLUSION

It is indisputable that non-uniformities of stress and strain develop in samples of soil tested in the two available simple shear apparatus. It is, however, not necessary to dismiss these apparatus since meaningful results, regardless of the plasticity of the soil, on the mechanical behavior of soils under monotonic loading can be obtained by measuring the stresses and pore water pressures on the sample core.

It was shown in this paper that elastic analyses tended to produce larger stress concentrations at the ends of the sample in the plane of shear deformation than analyses based on the assumption that the behavior of the soil specimens can be modelled by the modified Cam-clay model. The experimental evidence suggests that the stress-strain behavior of very loose 14/25 Leighton Buzzard sand and speswhite kaolin from constant load simple shear tests is satisfactorily predicted by the modified Cam-clay model. This is, however, not the case for constant height tests. Even though the vertical effective stress is reasonably well predicted by the finite element program for kaolin, the shear stresses are overpredicted by about 50 percent.

Experimental data from the sample core of good quality tests are needed to validate the supposition that the changes in vertical stresses in constant height tests are equivalent to the changes in pore water pressures in constant load undrained tests in spite of the good agreement obtained from the numerical analysis.

ACKNOWLEDGEMENT

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NOTATION

p=mean effective normal stress
q=deviatoric stress
u=pore water pressure
E=elastic modulus
e_λ=void ratio at unit mean effective stress on critical state line
G=shear modulus
M=slope of critical state line
S_{ij}=stress deviator tensor
γ=engineering shear strain
δ_{ij}=Kronecker delta
κ=slope of swelling line
λ=slope of consolidation line
μ=Poisson's ratio
σ_{ij}=effective stress tensor
$\sigma_1, \sigma_2, \sigma_3$=effective major, intermediate and minor principal stresses
σ_a=effective applied vertical stress
$\sigma_x, \sigma_y, \sigma_z$=effective normal stresses in x, y and z directions respectively
τ_{yx}=shear stress
ϕ=effective angle of internal friction
ϕ_{cv}=effective angle of internal friction at the critical state

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